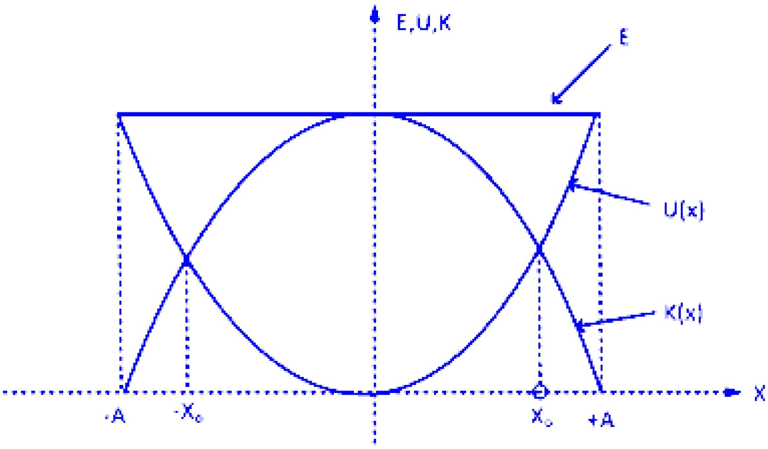
Simple Harmonic Motion

Question1

The variations of kinetic energy $K(x)$, potential energy $U(x)$ and total energy as a function of displacement of a particle in SHM is as shown in the figure. The value of $|x_0|$ is



KCET 2025

Options:

A. 2 A

B. $\frac{A}{\sqrt{2}}$

C. $\sqrt{2}$ A

D. $\frac{A}{2}$

Answer: B

Solution:

To understand the displacement x_0 where kinetic energy (KE) equals potential energy (PE) in simple harmonic motion (SHM), consider the following explanation:

At the position x_0, the kinetic energy is equal to the potential energy. Mathematically, this is represented by:

$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2$

Here, the left side of the equation represents the kinetic energy, and the right side represents the potential energy. Simplifying the equation:

$A^2 - x^2 = x^2$

Rearranging terms, we get:

$A^2 = 2x^2$

Solving for x:

$x^2 = \frac{A^2}{2}$

$x = \frac{A}{\sqrt{2}}$

Thus, the value of $|x_0|$ is $\frac{A}{\sqrt{2}}$.

Question2

For a particle executing simple harmonic motion (SHM), at its mean position

KCET 2024

Options:

A. velocity is zero and acceleration is maximum.

B. velocity is maximum and acceleration is zero.

C. both velocity and acceleration are maximum.

D. both velocity and acceleration are zero.

Answer: B

Solution:

We know that velocity v and acceleration of a particle in SHM.

$v = \omega\sqrt{A^2 - y^2}$ and $a = -\omega^2 y$

$\therefore$ At mean position, $y = 0$

$$\therefore \quad v = \omega\sqrt{A^2 - 0}$$
$$\Rightarrow \quad v_{\max} = \omega A \text{ and } a = 0$$

Question3

A block of mass m is connected to a light spring of force constant k. The system is placed inside a damping medium of damping constant b. The instantaneous values of displacement, acceleration and energy of the block are x, a and E respectively. The initial amplitude of oscillation is A and ω' is the angular frequency of oscillations. The incorrect expression related to the damped oscillations is

KCET 2023

Options:

A. $x = Ae^{-\frac{b}{m}} \cos\left(\omega' t + \phi\right)$

B. $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

C. $E = \frac{1}{2}kA^2e^{-\frac{bt}{m}}$

D. $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

Answer: A

Solution:

Equation of motion for damped oscillation is $\frac{md^2x}{dt^2} + \frac{bdx}{dt} + kx = 0$

Angular frequency is $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Displacement is given by, $x = A_0e^{-\frac{b}{2m}t} \cos(\omega t + \phi)$

and energy is given by $E = \frac{1}{2}kA^2e^{-\frac{b}{m}t}$

Question4

The displacement of a particle executing SHM is given by $x=3\sin\left[2\pi t+\frac{\pi}{4}\right]$, where x is in metre and t is in seconds. The amplitude and maximum speed of the particles is

KCET 2022

Options:

A. $3\text{ m}, 4\pi\text{ms}^{-1}$

B. $3\text{ m}, 6\pi\text{ms}^{-1}$

C. $3\text{ m}, 8\pi\text{ms}^{-1}$

D. $3\text{ m}, 2\pi\text{ms}^{-1}$

Answer: B

Solution:

Given, displacement equation of particle executing SHM,

$X=3\sin\left[2\pi t+\frac{\pi}{4}\right]$

Comparing with displacement equation of SHM as $y=A\sin(\omega t+\phi)$, we get

Amplitude, $A=3\text{ m}$

Angular velocity, $\omega=2\pi\text{rad/s}$

$\therefore$ Maximum speed,

$v_{\max}=\omega A=2\pi\times 3=6\pi\text{m/s}$

Question5

A pendulum oscillates simple harmonically and only if

I. the sizer of the bob of pendulum is negligible in comparison with the length of the pendulum.

II. the angular amplitude is less than 10°.

Choose the correct option.

KCET 2021

Options:

A. Both I and II

B. Only I

C. Only II

D. None of these

Answer: A

Solution:

A simple pendulum in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string.

It will oscillates simple harmonically, only if

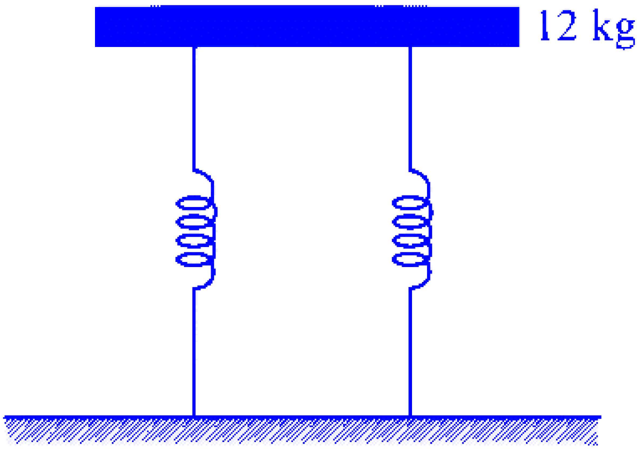
(I) the size of the bob is negligible as compare to the length of the string of pendulum.

(II) the angle between the normal drawn at the mean position and string at extreme point or angular amplitude is less than 10°.

Thus, both the statements are correct.

Question6

A tray of mass $12 \mathrm{~kg}$ is supported by two identical springs as shown in figure. When the tray is pressed down slightly and then released, it executes SHM with a time period of $1.5 \mathrm{~s}$. The spring constant of each spring is



KCET 2020

Options:

A. 50 Nm^{-1}

B. 0

C. 105 Nm^{-1}

D. ∞

Answer: C

Solution:

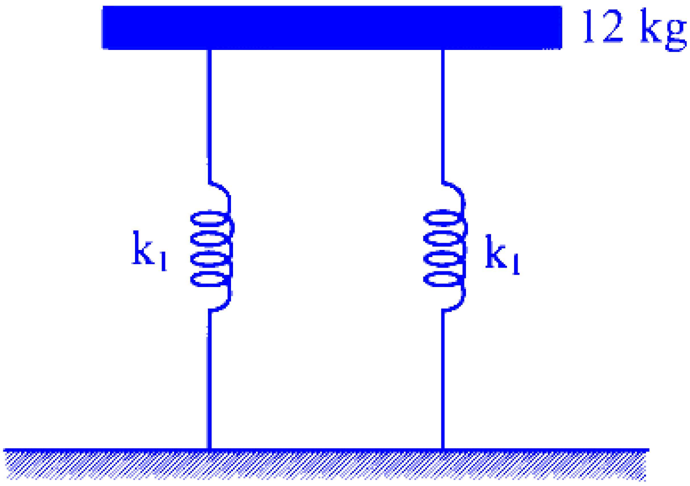
Mass of tray, $m = 12$ kg

Time period, $T = 1.5$ s

If k be the spring constant of each spring, then

$k_1 = k_2 = k$

since, springs are connected in parallel, hence



$k_{\text{baA}} \quad = k + k$

Time period, $T = 2\pi\sqrt{\frac{m}{k_{\text{net}}}} = 2\pi\sqrt{\frac{m}{k_1+k_2}}$

$\Rightarrow \quad 1.5 = 2\pi\sqrt{\frac{12}{k+k}} \Rightarrow 1.5 = 2\pi\sqrt{\frac{12}{2k}}$

$\Rightarrow \quad 1.5 = 2\pi\sqrt{\frac{6}{k}}$

Squaring both side, we have

$$2.25 = 4\pi^2 \cdot \frac{6}{k}$$

$$\Rightarrow \quad k = \frac{4\pi^2 \times 6}{2.25}$$

$$= 105.17 \text{ Nm}^{-1} \approx 105 \text{ Nm}^{-1}$$

Question7

A piston is performing S.H.M. in the vertical direction with a frequency of 0.5 Hz. A block of 10 kg is placed on the piston. The maximum amplitude of the system such that the block remains in contact with the piston is

KCET 2019

Options:

A. 1 m

B. 0.5 m

C. 1.5 m

D. 0.1 m

Answer: A

Solution:

Given, frequency of SF, $v = 0.5$ Hz

mass of block, $M = 10$ kg amplitude, $A = ?$

we know, $a_{\max} = \omega^2 A$ ($a_{\max}$ = maximumacceleration ω - angular frequency. A = amplitude)

for block to remain in contact with piston $F_{\max}$ = contact. of block $= Mg; Mg = M\omega^2 A$

$A = \frac{g}{\omega^2} = \frac{9.8}{(2\pi v)^2} = \frac{9.8}{(2 \times 3.14 \times 0.5)^2} = 1 \text{ m} (\because \omega = 2\pi v)$

Question8

Two simple pendulums A and B are made to oscillate simultaneously and it is found that A completes 10 oscillations in 20 sec and B completes 8 oscillations in 10 sec . The ratio of the length of A and B is

KCET 2017

Options:

A. $\frac{25}{64}$

B. $\frac{64}{25}$

C. $\frac{8}{5}$

D. $\frac{5}{4}$

Answer: B

Solution:

To determine the ratio of the lengths of pendulums A and B, we start with the given information:

For pendulum A:

It completes 10 oscillations in 20 seconds.

The time period T_A is the total time divided by the number of oscillations:

$$T_A = \frac{20 \text{ seconds}}{10} = 2 \text{ seconds}$$

The formula for the period of a pendulum is given by:

$$T_A = 2\pi\sqrt{\frac{L_A}{g}}$$

where L_A is the length of pendulum A and g is the acceleration due to gravity.

For pendulum B:

It completes 8 oscillations in 10 seconds.

The time period T_B is:

$$T_B = \frac{10 \text{ seconds}}{8} = 1.25 \text{ seconds}$$

The formula for the period of pendulum B is:

$$T_B = 2\pi\sqrt{\frac{L_B}{g}}$$

where L_B is the length of pendulum B.

To find the ratio of their lengths, we need to compare the periods using the formula for the period:

Step 1: Express the square of the periods based on their formula:

$$T_A^2 = \left(2\pi\sqrt{\frac{L_A}{g}}\right)^2 \quad \Rightarrow \quad \left(\frac{L_A}{g}\right) = \left(\frac{T_A}{2\pi}\right)^2$$

$$T_B^2 = \left(2\pi\sqrt{\frac{L_B}{g}}\right)^2 \quad \Rightarrow \quad \left(\frac{L_B}{g}\right) = \left(\frac{T_B}{2\pi}\right)^2$$

Step 2: Divide the two expressions to find the ratio $\frac{L_A}{L_B}$:

$$\frac{L_A}{L_B} = \left(\frac{T_A}{T_B}\right)^2$$

Substitute the values of T_A and T_B:

$$\frac{L_A}{L_B} = \left(\frac{2}{1.25}\right)^2 = \left(\frac{160}{100}\right)^2 = \frac{64}{25}$$

Thus, the ratio of the lengths of pendulum A to B is $\frac{64}{25}$.
